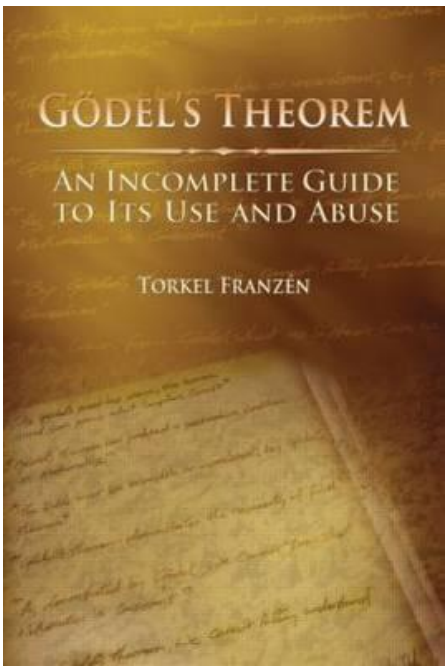

Gödel's Theorem: An Incomplete Guide to Its Use and Abuse

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Among the many expositions of Gödel's incompleteness theorems written for non-specialists, this book stands apart. With exceptional clarity, Franz n gives careful, non-technical explanations both of what those theorems say and, more importantly, what they do not. No other book aims, as his does, to address in detail the misunderstandings and abuses of the incompleteness theorems that are so rife in popular discussions of their significance. As an antidote to the many spurious appeals to incompleteness in theological, anti-mechanist and post-modernist debates, it is a valuable addition to the literature." --- John W. Dawson, author of "Logical Dilemmas: The Life and Work of Kurt G del

Gödel's Theorem: An Incomplete Guide to Its Use and Abuse Details

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From Reader Review Gödel's Theorem: An Incomplete Guide to Its Use and Abuse for online ebook

Cait says

So far I'm *enjoying* this book but not *respecting* it. For a book intended as a non-mathematician's guide to Gödel's work, it reads like many a textbook I've had. I personally *like* that -- I find theorems on prime numbers very soothing -- but it leaves me suspicious of the book's self-awareness.

I'll have to pull up some quotes as I go....

Babak says

An elegant book on clarifying the Gödel's incompleteness theorems, and their margins. It also well clarifies some of the common misconceptions on these theorems.

However and contrary to what is somewhat implied in introduction, the content is compact! and one can't understand it without picking up a pen and a piece of paper, and opening the details of the explained subjects by oneself.

Morgan says

This book is a good introduction to what Godel's theorem is, and what it is not. It gives a lot of examples of how people have used the theorem, and discusses why only some of those uses are correct. The book is moderately technical. It gives enough information to understand the mathematics of the theorem, but only sketches various proofs.

The organization of the book is a bit cumbersome. Computability and Godel numbering, for example, are used extensively in chapter 2 but aren't explained fully until chapter 3. The author motivates this choice by saying that you only need a superficial understanding at first, and that deeper explanations can follow. Nonetheless, I found this confusing each time it happened.

After reading this book, I have a much better handle on the ideas of completeness and consistency of formal systems. I also understand how computability plays into the incompleteness theorem. This is definitely an informative book.

The second half of the book goes over various uses that people have made of Godel's incompleteness theorems, and how those uses often don't have anything to do with the actual theorem. It also goes over some ways that Godel's theorem is related to Kolmogorov Complexity and Turing machines. The author is primarily responding to statements made by other people, and his responses are sometimes interesting and sometimes just tedious.

Barry says

Having had more classes than I care to remember in various aspects of computational theory (and trust me, I'm fast forgetting all of them), I've been exposed to Turing machines and the Incompleteness Theorem. I may have even on occasion said something to the effect that such-and-such was not possible due to said theorem. No more.

This book makes Godel about as approachable as it can be made. This is to say that it's certainly possible to sit down and read it over a good spaghetti dinner and think that you're understanding it, only to pay the check and realize that, if called upon to give a short summary, you probably couldn't. As much fun as this book is to read, it really demands a quiet library and some scratch paper for notes. But don't let this deter you from taking it with you to dinner -- it's also a book that deserves to be read multiple times.

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g.low_koma says

From one of the most suitable angle, this book can be taken as a pretty well written, detailed, and temperate defusing-vademecum whose aim is, among other things, to provide an useful diorama for the *overschooled pundits*, later postmodernists, and Deleuzian fanboys who apparently are still versed in bamboozling themselves into **special relativity-**, **QM-**, and obviously **Gödel-**babbling (e.g. that his theorems would enact any sort curb, or an incompleteness advice to the limits of mathematics and science, &so on &so forth). It turns out to be excellent, fully on the mark, and rewarding to read; Franzén does indeed a pretty valuable job especially to expounding how the theorems do not eclipse some supposedly, and once adamantine, concept of Logical Truth - how this was not its reach at all or its scientific heritage, and how it *concretely* applies solely at the axiomatic level (as an equivalent for the impossibility of an algorithm to decide whether _all_ true arithmetical proposition are... true), and let alone that it is not even that 'normal' for a science to posit on axiomatic formulations. To borrow from Hintikka: *the theorem casts absolutely no shadow on the notion of truth*.

Given the book is neither so *entry level* (despite being full of quite good verbal panning) as it would require a little intuitive understanding of set theory and some algebra, nor a highbrow lit sensation as any trifle out of Hofstadter's hands could be - the ones who may need it the most, alas, won't ever manage to grasp it.

Isk says

Unfinished. The math isn't well-explained, and it's annoyingly obvious that the abuses of Goedel's Theorem are abuses.

Kevin K says

Important: Contrary to the claims, THIS BOOK IS NOT WRITTEN FOR GENERAL READERS OR NON-MATHEMATICIANS. I myself have a math degree and found the book quite challenging. A person with

little math experience will be in for a very tough slog.

That said, this is a concise and well-written overview. It covers Godel's two incompleteness theorems, and a number of important related topics (e.g., the completeness theorem, compactness theorem, non-standard analysis, large cardinals). The level of detail was just about right for a reader with math experience; enough to understand the principles of the proofs, without too much of the technical machinery.

Best of all was the debunking. I have long felt that Godel's theorem is overhyped, and it was wonderful to read Franzen's takedowns of a number of half-baked arguments (Penrose and Chaitin in particular). In my opinion, Godel's theorems bear a strong resemblance to Anselm's ontological proof of the existence of God. In both cases, a slick piece of *a priori* logic seems to have huge implications in the real world. I don't trust such arguments. To me they smack of scholasticism. It's as though people were to invest the Liar's Paradox ("this sentence is false") with some deep significance, and spend decades arguing about its applications to the real world. The blunt truth is that the Liar's Paradox (and Godel's theorems, which closely resemble it) have no applications in the real world. My theory: we will learn about consciousness and the brain the hard way, by actually studying it empirically, not by divining its secrets with fascinating, but ultimately trivial, bits of logic.

Leo Horovitz says

I think my first real encounter of a clear abuse of Gödel's incompleteness theorem came when I was engaged (as I so often am) in the debate on religion, online as well as elsewhere. This was one of the former kind and in one of the lower subcategories of the bigger category of online venues for the exchange of ideas: YouTube... Some atheist or number of atheists had argued against religion, presumably (because the response regarded this aspect of the religious question, but it wouldn't surprise me much to learn that the atheist/atheists in question had in fact asked about the ethical standards of the Bible or something else completely unrelated, the intellectual integrity and rational capacity of the staunch Bible defenders most of the time leave something to be desired) specifically regarding the question of the rationality behind belief in god. The response went something along the lines of this: "Gödel proved that there are unknown/unprovable truths [he did nothing of the sort], and therefore... [something about how belief in phenomena without evidence isn't so crazy after all]". The whole thing was topped off with the brilliant argumentative tactic consisting in showing a photograph of Gödel standing next to Einstein and saying something like "Look what kind of friends he had! Kind of a smart guy that Einstein!" This feeble attempt got some responses of its own pointing out how this application of Gödel's theorem to a religious debate was... hrm, somewhat misguided (for an offense to reason of this magnitude, any adjective seems insufficient so why not use one that is so wildly insufficient as to call attention to the difficulty of finding the proper words to describe how bad it is?), though as I recall, the commenters, quite appropriately, used much harsher words. Regarding those who abused Gödel in such a horrible fashion in this particular instance, I hold little hope as to their ability to understand either the theorem itself or the actually fairly simple arguments needed to explain why it was not applicable to this situation, but in the hope that such attempts are not always entirely vain, here's a book clarifying the issues!

Franzén has written an overview of the whys and hows of Gödel's theorem in a general, fairly non-technical way, so that one can see what exactly the theorem states, what it does *not* state (which we will focus on a little more soon), when and where it is applicable and what general conclusions can be drawn from it on purely mathematical grounds (which *does* include considerations in philosophy of mathematics as these are still bound by the technical details of the mathematics, but does *not* include metaphorical extensions of the

theorem into other fields in ways that take *no* considerations of the mathematics needed to prove the theorem). The focus of the book is to guide the reader through the landscape of the theorem in order to show when it can be called upon and when it can not by exhibiting a number of bad arguments that try to lean on the theorem, but fail to produce convincing (or in many cases, even sensical) arguments for their conclusions due to failures to understand the theorem, misconceptions about its reaches and consequences, and showing how these arguments go wrong. The examples are gathered into different categories: there are those that, as in my anecdote above, relate to religious debates (but the examples found in the book tend to be more about atheists misapplying the theorem to try to show how the Bible is necessarily "incomplete" if "consistent", a grave error resulting from a misunderstanding of the theorem though the Bible is quite obviously "incomplete" in the sense of not being a complete guide to the universe as some Christians like to claim it is, but not an error showing a misunderstanding of the theorem anywhere near the level exhibited by the Christians on YouTube described above), to arguments against the possibility of a Theory of Everything in physics, bold proclamations of a "post-modern" era in mathematics following Gödel's theorem (!), and various arguments about the implications of the theorem both for the supposed limits of the human mind to grasp all truths and for the limits of *computers* as contrasted against human minds concerning their ability of proving theorems.

None of these attempts at deriving interesting conclusions in other fields from the proof of Gödel's theorem follow at all in fact. The main points Franzén use to show this is that there is a requirement upon any system exhibiting incompleteness *first* that it is a formal system (in a technical sense meaning that it is a set of basic axioms with a set of rules for deriving new theorems), *second* that it is possible to do a certain minimal set of basic arithmetic in the system (a requirement that needs no further elaboration here other than noting that this requirement is specified exactly), and *third* that even when a system *does* exhibit these characteristics, the incompleteness is still only a property of the arithmetical component of the system, the theorem says *nothing* about the completeness or incompleteness of the system with regards to the other, non-arithmetical, statements included in it. Taking this as a starting point for evaluating claims of incompleteness found in other areas, or supposed philosophical implications of the theorem (of which there are many that are legitimate), Franzén goes on to show what kind of misunderstandings that seem to lie behind the abuses of this famous theorem.

We need not let the details of Franzén's investigation prolong this text needlessly, but some deliberation on the kind of things people have been found to claim in regards to the theorem deserve mention. Many seem to think that Gödel showed that the peculiar sentence "This statement is not provable in the theory PA" while shown to be indeed, not provable in PA, has somehow still been shown to be true and that it has been shown to be true in some fashion that goes beyond formalization. The view seems to be based on the observation that this formal theory has been shown to not be able to prove the statement whereas this fact is just what the statement says so we can see that it is true after all. So far so good (unless I'm misrepresenting things a bit here myself, I have to be careful not to do what the people exhibited in the book are called out for doing): Gödel *did* show the statement to be unprovable and if it is indeed unprovable, it is true since this is what it says. The problem with overinterpreting this is that *if* we *do* see that it is true, we do so by some further deliberation, probably by evaluating the claim and seeing that the statement's unprovability in PA is just what makes it true. This further deliberation is necessary to see this though, and Gödel's theorem by itself by no means shows that the statement is unprovable but true (the only way it could be shown to be true by a proof carried out in the system PA is if it could be proven in PA, which is exactly what can not be accomplished). So no conclusions regarding "true but unprovable statements" follow: the statement is unprovable in PA, not in any absolute sense (there *is* no absolute sense of being provable). Another source of confusion is that people generally fail to understand that Gödel proved only that *if* the theory in question is consistent, *then* it is incomplete, not that it *is* incomplete. So the question of the incompleteness of the theory depends on whether it is consistent or not, but this is something which, according to the second

incompleteness theorem, the theory itself *cannot* prove if it *is* consistent. In other words, with additional reasons to suppose the theory to be consistent, we can draw the conclusion that it is indeed incomplete, but we can not prove it to be so unless we can also prove the theory's consistency, which needs another theory which will itself be incomplete if consistent and unable to prove its own consistency if consistent and so on. This all complicates matters to a degree rarely taken into account in the many attempted uses (turned *ab*uses) of the theorem.

Franzén does an excellent job exhibiting some common (and perhaps some not so common but nevertheless sever and therefore, attention worthy) abuses and explaining carefully why they *are* abuses. In doing so, he also covers the landscape of related results, additional ways to prove incompleteness that do not rely upon Gödel's strange self-referential formula showing, importantly, that the theorem is *not* just something having to do with self-referential (always a suspect in intellectual discourse) exotic sentences never encountered outside the proof of the theorem. Though doing so is necessary to understand the theorem thoroughly enough to appreciate who and why uses of it go wrong, but Franzén tends to take these mathematical side stepping too far, going into the many (interesting but nonetheless inessential to the question of the abuses of the theorem) details of the theorem and its implications (even when it is done in a mostly informal fashion) does little to inform the reader of why many popular attempts to draw conclusions from the theorem go wrong, and the facts that this does little to inform the reader in this is evident in how Franzén uses these issues in exhibiting the failures in the abuses: not much at all. Again and again, when he demolishes yet another piece of bad writing referring to Gödel, he comes back to the main points mentioned above: the theorem only applies to formal systems powerful enough to support a certain amount of arithmetic and even so, only to the arithmetical component. The additional details of different axioms for mathematics, variants of arithmetic and so on are of course very important for understanding the implications as well as the applications of the theorem, but are only relevant to a exposition of the abuses of the theorem when such details can be used to show the errors in the abuses, otherwise they belong instead in a much more encompassing work on the reaches and limits of Gödel's theorem, a work that would *not* be focused on explaining how and why so many of the attempts at using Gödel's theorem outside its field fail. Such a book would be extremely interesting, but it would need to be much, much, long than the current text which does seem to want to be about the abuses. It is also clear from the text that the abuses *are* in focus since the rest is just there to "set the stage" and asides into the land of mathematics not directly related to any abuses of Gödel's theorem only arrive when the discussion slides into them. In these cases, Franzén should have backed off a bit more readily and kept his focus on exhibiting abuses which would, on some occasions, have been more interesting and enlightening had there been a few more pages devoted to them.

Another problem with Franzén's willingness to take up so many related issues regarding the theorem is that the reader can easily be overwhelmed by all the terminology in a book that is, after all, written for non-experts. It's even claimed to be accessible to people with no previous background in logic, a claim I'm by no so tired of commenting upon that I mostly force myself to do so due to some sense of obligation: it's *technically* (though not in the mathematical sense of course) true that no previous knowledge in logic is required since every bit of terminology needed to understand the argumentation is defined in the book, but considering the number of such definitions, any reader not already at least familiar with logic and Gödel's theorem is bound to be confused fairly quickly. Franzén *does* do a good job of commenting upon when a certain technicality is essential for understanding the rest and when it is not, but this is hardly sufficient since any reader not already familiar with the terminology will probably fairly quickly lose track of which of these terms he or she needs to remember and which only appears parenthetically. These kind of claims of the lack of a requirement of previous knowledge are so common in logic texts (and I suppose in other areas as well though I suspect it's somewhat peculiar to formal science where such claims seem necessary as to not scare away potential readers) that I've gotten used to it, but I still feel the aforementioned duty to report on them.

It is in any case a very good book and, as far as I understand, a very original one. It does an excellent job of showing how Gödel's theorem can be abused and how to respond to such abuses, but it is not the best choice for an introduction to the theorem or it's implications. It is not primarily a guide (or at least not among the best of those) to what the theorem *does* mean but what it does *not* mean.

Zoe Jackson says

A boon for anyone who's read a populist account of Gödel's incompleteness theorems and been left with a vague sense of unease from the amount of technical detail glossed over, and the sweeping claims made for the theorems' consequences. This book, if given close attention, will reward even the lay-reader with a precise, technical working of the proofs and their proper (and surprisingly limited) spheres of application. Spoiler alert: anyone who tries to use these theorems to argue for the relativity of truth or the mind being more than the brain is hopelessly off-track, and even the most respected mathematicians and physicists (Hawking, Penrose, Chaitin, etc) are not immune from making such mistakes.

Joseph says

Neat little book, it helped dispel some of my own misconceptions around
Gödel's work.

It's definitely not a simple book to read, you'll need to dig in an grapple with the logical, algebraic language. It doesn't presuppose that you understand a lot of math, but people who have done a lot of math are a lot more used to reading a lot of dense sentences, so they'll have an easier time.

Once you can get through the in-depth logical stuff, you'll reach the pretty obvious takeaway: that an unfortunate number of people view
Gödel's theorems, not just as a metaphor for work in their own fields, but as somehow "proving" a lot of very silly things based on a conflation of technical and colloquial meanings of words like "formal" or "system". Essentially, it's like the obligatory XKCD:

Still, if you can stomach the math, it's quite a good book, and a good reminder to make sure you damn well know what you're talking about.

William Ramsdell says

I found this an excellent introduction for the SEMI-technical reader. It is not, as it claims, fit for completely non-technical persons. The organization could be improved slightly, but again, this hit an almost perfect chord with me and my preexisting knowledge of the subject (I've taken 2 courses in Logic, taught logic, understand various forms of rudimentary logical proof, etc).

Muhammad al-Khwarizmi says

Can't give this book two stars. I'd like to give it 2.5. Franzén explains some of the ways Gödel's theorems are misused well enough but his mathematical prose is confusing. I've read much clearer expositions of the same / similar topics.

Charles says

Godel's Incompleteness Theorems were a revolution in mathematics and there were repercussions and misunderstandings that rippled out into other fields. The main theorem first appeared in an Austrian journal in 1931 and can be stated very simply.

In any consistent formal system S within which it is possible to perform a minimum amount of elementary arithmetic, there are statements that can neither be proved nor disproved.

The consequences are enormous, in that it means that in any system that can be used to perform arithmetic, there will be theorems that can never be verified as either true or false. In other words, some knowledge will forever be unattainable within that system. Of course, this does not preclude adding additional axioms that will allow other theorems to be proved.

Franzen does an excellent job in explaining the incompleteness theorems in a manner that can be understood by people with a limited knowledge of mathematics. While there are few places where a high school mathematics education is not sufficient to understand a more technical argument, it will be enough to understand and appreciate the theorems.

My favorite parts of the book were the sections devoted to "applications" of the incompleteness theorem outside of mathematics. Some examples are from religion, political science and philosophy. Godel's theorems are used to "prove" that no religion can contain a complete set of answers and that any constitution must of necessity be incomplete. Human thought is also interpreted in the context of the incompleteness theorems. The statement is:

Insofar as humans attempt to be logical, their thoughts form a formal system and are necessarily bound by Godel's theorem.

This statement and others related to the nature of human thought are examined in detail. The philosophy of Ayn Rand is also examined as a system that must of necessity be incomplete. This book would be an excellent supplemental text for a philosophy course where the nature of truth is examined. It would also be a very good choice for a course in the philosophy of mathematics.

Published in Journal of Recreational Mathematics, reprinted with permission and this review also appears on Amazon.

Bill W says

This was probably the best “popular” mathematics book I’ve ever read. Prior to reading this book I had assumed that Goedel’s theorems would always be too esoteric for me to comprehend. Certainly my previous attempts were unsuccessful. Franzen’s book is amazingly clear and cogent, interesting and fun. The deconstructions of wrong applications of the theorems in both popular and technical literature are especially amusing and educational at the same time. I wish there were more books like this.

Ton van Gessel says

This book tells you what Godel's theorem really is about. It's not the easiest book to read if like me you don't have a formal math education. But if you take the time to read, reread, think about it en reread again you can eventually understand it (I could). Godel's theorem is mentioned in passing in many books. This book gives you the tools to check if it really has anything to do with Godel's theorem (to my surprise in many cases it doesn't).
